isopropylamine for each milliliter of aqueous solution, while for precipitating 90% of $KIO_3$ only 0.7 ml of isopropylamine is required (Mosseri and Alfassi, 1983). Thus, the following multistage process will be more economical.

Let us assume that we will add $V$ ml of MOS to 1 ml of the aqueous solution. The precipitated fraction, $f$, of the precipitable electrolyte $B$ is given by Eq. 1 as $f = k \ln V/V_c$. The next step will be distillation of MOS followed by evaporation of $f$ ml of the aqueous solution, which will lead to precipitation of fraction $f$ of the other electrolyte, $A$, leaving $(1 - f)$ ml of solution similar to the mother liquor, which can be treated again or returned to the original solution. The total volume of MOS required to precipitate all $B$, $V_{\text{total}}$, is given by

$$V_{\text{total}} = V + (1 - f)\cdot V + (1 - f)^2 V + \ldots = V/f \qquad (2)$$

Substituting Eq. 1 for $f$ gives

$$V_{\text{total}} = V/(k \ln V/V_c) \qquad (3)$$

Differentiating $V_{\text{total}}$ with respect to $V$ and equating to zero lead to $\ln V/V_c = 1$, or

$$V_{\min} = eV_c \qquad (4)$$

where $e$ is the base of the natural logarithm and $V_{\min}$ is the amount added in each step per 1 ml of aqueous solution in order to assure minimum use of MOS; the amount precipitated in each step is $k$. Thus, for precipitation of $KIO_3$ with isopropylamine where $V_c = 0.028$ and $k = 0.251$, it will be most economical to add in each step 0.076 ml of isopropylamine per 1 ml of aqueous solution, which will lead to precipitation of 25% of the $KIO_3$.

In the case $V_{c_A} \gg V_{c_B}$ we saw that the one-step process is feasible, although less economical than a multistep process. However, if the system fulfills only $V_{c_A} > V_{c_B}$ such that 99% precipitation of B already involves some precipitation of A, then only multistep separation is possible. Any $V$ that fulfills $V_{c_A} > V_{c_B}$ may be used in each step. If $V_{c_A} > e\cdot V_{c_B}$, it goes back to the situation described in the previous section where the same multistep process was suggested also for $V_{c_A} \gg V_{c_B}$. A characteristic case can be the separation of NaCl and NaBr by the addition of acetone, since $V_c$,NaCl = 0.051 and $V_c$,NaBr = 1.649 (Alfassi and Ata, 1983).

In the case

$$V_{c_A} \simeq V_{c_B} \quad \text{but} \quad k_B > k_A$$

the separation can be done in a multistep process, since the precipitation with MOS will lead to precipitated fraction enriched with respect to $B$, while the fraction remaining in solution is enriched with respect to $A$.

## NOTATION

$f$ = fraction of the electrolyte precipitated by MOS
$k$ = solventing-out constant
$V$ = MOS's volume per 1 ml aqueous solution
$V_c$ = minimal value of $V$ that brings to precipitation all of the electrolyte
$V_{\text{total}}$ = total volume of MOS required to precipitate all the electrolyte in 1 ml aqueous solution in a multistep process
$V_{\min}$ = minimal value of $V_{\text{total}}$.

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# Direct Contact Heat Transfer with Change of Phase: Theoretical Model Incorporating Sloshing Effects

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Heat transfer to dispersed liquid drops vaporizing in a continuous immiscible and low volatile liquid medium has extensive applicability in a number of engineering fields. One of the pressing problems in recent years is the application to power generation utilizing geothermal energy (Boehm et al., 1974) by means of binary cycles of refrigerants. Another representative application is the saline and wastewater treatments in which such solutions are separated into pure ice crystals and residual enriched liquids.

The objective of the present work is to extend the authors' (1982) previous theoretical analysis by considering the effect of sloshing. This effect is nonexistent for the case of small drops (less than 2 mm dia.), evaporating through high-viscosity continuous liquid phase. For the case of small drops, the rigid drop model is suitable, and the equation for the heat transfer coefficient is given by the authors (1982) as

$$h_c = 0.314 \frac{K_c}{R} \left[\frac{UR}{\alpha_c}\right]^{1/3} \left[(\beta - \gamma) - \frac{1}{2}(\sin 2\beta - \sin 2\gamma)\right]^{2/3} \qquad (1)$$

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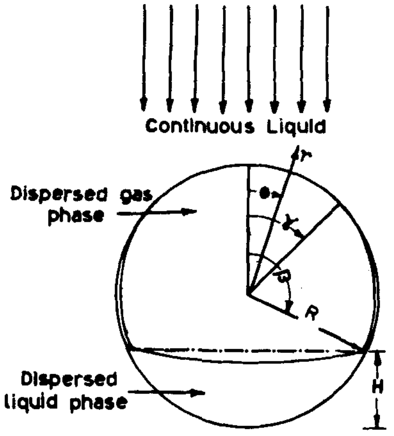

Figure 1. Configuration of two-phase bubble.

| Models | Symbols |
|---|---|
| Raina, Grover Eq. 1 | —— |
| Sideman, Isenberg Eq. 27 | —·—·— |

When the combined action of surface force balance and sloshing is taken into account, the heat transfer coefficient is given by

$$h_c = 0.2916\frac{K_c}{R}\left[\frac{UR}{\alpha_c}\right]^{1/3}\left[\beta - \frac{\sin 2\beta}{2}\right]^{2/3} \qquad (2)$$

or, in terms of Nusselt number

$$Nu_c = 0.4629[Pe_c]^{1/3}\left[\beta - \frac{\sin 2\beta}{2}\right]^{2/3} \qquad (3)$$

which is derived as follows.

The equilibrium location of the end of the film formed by surface force balance is estimated from the following relation obtained by Mori (1978) for two-phase bubbles:

$$1.5\frac{\mu_c U}{S_d} = \frac{1}{\cos\gamma - \cos\beta} \qquad (4)$$

The energy equation in spherical coordinates is expressed as

$$\frac{\partial T}{\partial t} + u_r\frac{\partial T}{\partial r} + \frac{u_\theta}{r}\frac{\partial T}{\partial \theta} = \frac{\alpha_c}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) \qquad (5)$$

Equation 5 is analogous to that used by Baird and Hamielec (1962) for the mass transfer around a rigid sphere.

In quasi-steady state conditions, near the surface, Eq. 5 reduces to

$$Ru_r\frac{\partial T}{\partial r} + u_\theta\frac{\partial T}{\partial \theta} = R\alpha_c\frac{\partial^2 T}{\partial r^2} \qquad (6)$$

Assuming that the thermal boundary layer around a two-phase bubble is much thinner than either the hydrodynamic boundary layer or the bubble radius, the tangential velocity can be approximated as

$$u_\theta = \dot{u}(r - R) \qquad (7)$$

The stream function

$$\psi = \int_R^r Ru_\theta \sin\theta\, dr = \tfrac{1}{2}(r - R)^2\, \dot{u}R\sin\theta$$

To simplify,

let $Y = (2\psi)^{1/2}$

then

$$Y = (r - R)(\dot{u}R\sin\theta)^{1/2} \qquad (8)$$

Replacing $r$ by $Y$ as a variable in Eq. 6, we get

$$\frac{Y}{(\dot{u}R^5\sin^3\theta)^{1/2}}\frac{\partial T}{\partial \theta} = \alpha_c\frac{\partial^2 T}{\partial r^2} \qquad (9)$$

The sloshing of the unevaporated dispersed liquid in the two-phase bubble is expected to cause an extension of the thin film lying between the zenithal angles $\beta$ and $\gamma$ to an extent of reducing the half opening angle $\gamma$ (Figure 1) to zero.

The frequency of renewal of the film formed as a result of sloshing is high enough to avoid dry patch formation within the two-phase bubble at the leading portion. Under such a situation, the heat transfer to two-phase bubble is expected to take place through the thin film lying between angle $\beta$ and $\gamma = 0$. Further, this film being very thin, it is reasonable to assume that it offers negligible inside resistance.

Let

$$X = \int_0^\beta (\dot{u}R^5\sin^3\theta)^{1/2}d\theta = \int_0^\beta R^2(\dot{u}R\sin^3\theta)^{1/2}d\theta \qquad (10)$$

Therefore, Eq. 9 reduces to

$$Y\frac{\partial T}{\partial X} = \alpha_c\frac{\partial^2 T}{\partial Y^2} \qquad (11)$$

The boundary conditions for Eq. 11 in terms of $X$ and $Y$ are

$$T = T_\infty: \quad Y > 0, \quad X < 0$$
$$T = T_\infty: \quad Y = \infty, \quad X \geqslant 0$$
$$T = T_L: \quad Y = 0, \quad X \geqslant 0$$

The solution of Eq. 11 with the help of these boundary conditions takes the form:

$$\frac{T_\infty - T}{T_\infty - T_L} = 1 - \frac{1}{0.893}\int_0^Z e^{-Z^3}dZ \qquad (12)$$

where

$$Z = Y\,(9\alpha_c X)^{-1/3} \qquad (13)$$

The local heat flux $q_\theta$ is given by

$$q\theta = K_c\left(\frac{\partial T}{\partial r}\right)_{r=R} = K_c\left(\frac{\partial T}{\partial z}\cdot\frac{\partial Z}{\partial Y}\cdot\frac{\partial Y}{\partial r}\right)_{r=R} \qquad (14)$$

From Eqs. 8, 12, and 13, we get the following relations, respectively:

$$\frac{\partial Y}{\partial r} = (\dot{u}R\sin\theta)^{1/2} \qquad (15)$$

$$\frac{\partial T}{\partial Z} = \frac{(T_\infty - T_L)}{0.893}e^{-Z^3} = \frac{(T_\infty - T_L)}{0.893}\ \text{at}\ Y = 0 \qquad (16)$$

$$\frac{\partial Z}{\partial Y} = (9\alpha_c X)^{-1/3} \qquad (17)$$

Using Eqs. 15, 16, and 17 in Eq. 14, we have

$$q_\theta = 0.539\,K_c\,(T_\infty - T_L)(\alpha_c X)^{-1/3}(\dot{u}R\sin\theta)^{1/2} \qquad (18)$$

Hence the local outside heat transfer coefficient $h_{c\theta}$ is expressed as

$$h_{c\theta} = \frac{q\theta}{\Delta T} = 0.539\,K_c(\alpha_c X)^{-1/3}(\dot{u}R\sin\theta)^{1/2} \qquad (19)$$

The average outside heat transfer coefficient related to the total area of a two-phase bubble is calculated as

$$h_c = \frac{1}{4\pi R^2}\int_0^\beta h_{c\theta}2\pi R^2\sin\theta\, d\theta$$
$$= 0.2695\,K_c\alpha_c^{-1/3}\int_0^\beta (X)^{-1/3}(\dot{u}R\sin^3\theta)^{1/2}d\theta \qquad (20)$$

Assuming that the flow around two-phase bubble obeys Stokes's theory we have

$$\dot{u} = 1.5\frac{U\sin\theta}{R} \qquad (21)$$

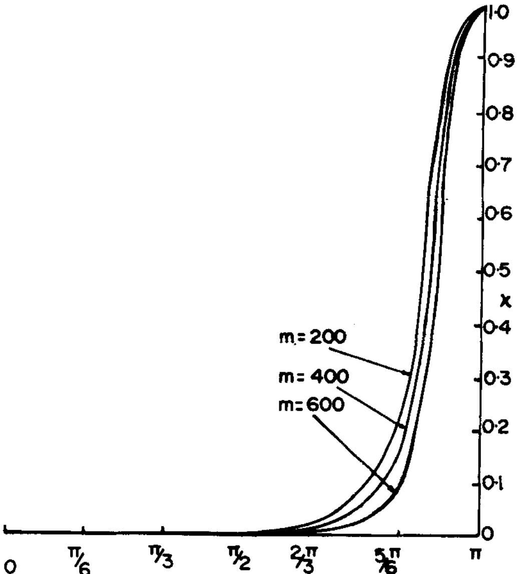

Figure 2. Half opening angle of bulk unevaporated dispersed liquid vs. vaporization ratio.

Using Eqs. 10, 19, and 21 in Eq. 20, we have

$$h_c = 0.2916\frac{K_c}{R}\left[\frac{RU}{\alpha_c}\right]^{1/3}\left[\beta-\frac{\sin 2\beta}{2}\right]^{2/3}$$

or, in terms of Nusselt number

$$Nu_c = 0.4629\,[Pe_c]^{1/3}\left[\beta-\frac{\sin 2\beta}{2}\right]^{2/3}$$

The value of angle $\beta$, involved in the above equation, is obtained as follows: The mass balance for constant mass bubble (Figure 1) yields

$$H^2(3R-H)=\frac{4(mR_o^3-R^3)}{(m-1)} \tag{22}$$

or, in terms of weight fraction vapor content $x$, we have

$$\frac{R^3}{R_o^3}=1+x\,(m-1) \tag{23}$$

$$\cos\beta=\frac{H-R}{R} \quad \text{or} \quad H=R\,(1+\cos\beta) \tag{24}$$

Substituting the value of $H$ from Eq. 24 in Eq. 22, we have

$$R^2\,[1+\cos\beta][3R-R(1+\cos\beta)]=\frac{4[mR_o^3-R^3]}{[m-1]}$$

or

$$(3\cos\beta-\cos^3\beta+2)=\frac{4}{(m-1)}\left[m\left(\frac{R_o}{R}\right)^3-1\right] \tag{25}$$

Substituting Eq. 23 in Eq. 25, we have

$$\begin{aligned}(3\cos\beta-\cos^3\beta+2)&=\frac{4}{(m-1)}\left[\frac{m}{1+x(m-1)}-1\right]\\&=\frac{4}{(m-1)}\left[\frac{(m-1)(1-x)}{1+x(m-1)}\right]\\&=\frac{4(1-x)}{1+x(m-1)}\end{aligned} \tag{26}$$

The calculated angle $\beta$ corresponding to the value of $x$ is shown in Figure 2. The experimental data of Sideman and Taitel (1964) are used for comparing the present modified expression Eq. 3 with the expression of Sideman and Isenberg (1967), as indicated in Figure 3. The modified expression shows a reasonable fit, in contrast with the value of $Nu_c$ obtained by Sideman and Isenberg (1967).

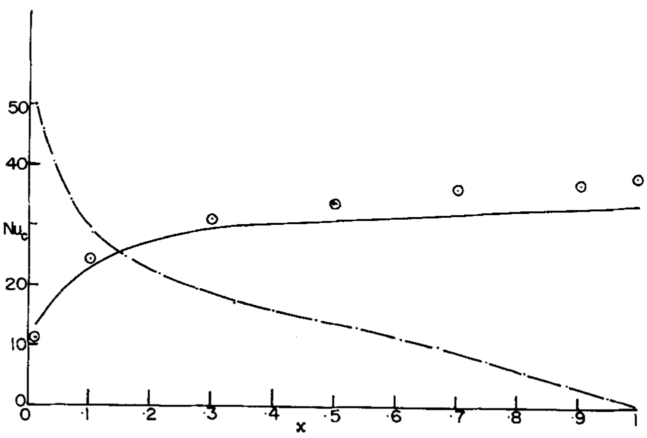

Figure 3. Comparison of models for heat transfer coefficient for pentane-water system.

| Models | Symbols |
|---|---|
| Sideman, Taitel Run, No. 16 | ⊙ |
| Sideman, Isenberg Eq. 27 | —·—·— |
| Present Model Eq. 3 | —— |

The value of $Nu_c$ obtained by Sideman and Isenberg (1967) has the form:

$$Nu_c = 0.977\,[Pe_c]^{1/2}\left[\cos\beta-\frac{1}{3}\cos^3\beta+\frac{2}{3}\right]^{1/2} \tag{27}$$

The experimental and theoretical points in Figure 3 are calculated on similar lines as mentioned by the authors in their previous paper (1982). The present model, with its simple form and easily calculable quantities, is also in fair agreement with the experimental data for small drops of less than 2 mm diameter.

During the early stages of evaporation, angle $\gamma = 0$ regardless of the size of initial drop diameter and the continuous phase viscosity. It is thus concluded that during evaporation, as the value of the unevaporated dispersed liquid decreases, sloshing takes place to provide increased surface area within the gas bubble for heat to liquid from the continuous liquid phase.

## NOTATION

$K$ = thermal conductivity, kW/m·K
$m$ = ratio of liquid to vapor density of the bubble
$r$ = radial distance in spherical coordinates, m
$R$ = radius of two-phase bubble, m
$R_o$ = initial radius of the dispersed liquid drop, m
$S$ = spreading coefficient, N/m
T = temperature, K
$T_L$ = interface temperature between the continuous and dispersed phases, K
$T_\infty$ = bulk temperature, K
$U$ = rise velocity of two-phase bubble, m/s
$\dot{u}$ = velocity gradient, L/s
$u_\theta$ = tangential velocity, m/s
$x$ = weight fraction vapor
$\mu$ = dynamic viscosity, N·s/m$^2$
$\alpha$ = thermal diffusivity, m$^2$/s

### Dimensionless Numbers

$Nu_c$ = Nusselt number, $\frac{h_c 2R}{K_c}$
$Pe_c$ = Peclet number, $U\cdot 2R/\alpha_c$

### Subscripts

$c$ = continuous phase
$d$ = dispersed phase

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# Gas Absorption Across Free Surface of a Stirred Vessel

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Owing to its industrial importance, gas absorption into a turbulent liquid in stirred vessels has been widely studied (Kozinski and King, 1966; Sada et al., 1979; Davies and Lozano, 1979; Kataoka and Miyamchi, 1972). As a result, many physical models were proposed to predict the rate of mass transfer across the free gas-liquid interface (Danckwerts, 1951; Davies, 1972; King, 1966; Lamont and Scott, 1970; Levich, 1962). One common basis of these models is that the turbulent eddies of Prandtl size or larger determine mass transfer rates at a free surface. Davies and Lozano (1979), through their measurements of turbulence close to the air-water interface, provided some positive experimental evidence.

However, it was felt that a firmer confirmation would be desirable since the results of Davies and Lozano showed scatterings and, in particular, the slope of the experimental curve $k_c/\sqrt{D}$ vs. $N$, which helped in drawing the conclusion, was not in good agreement with those estimated from various theories. The purpose of our work is to attempt a simple experimental means by which the validity of the conclusion of Davies and Lozano can be further solidified.

The theoretical background of this work is simple. Based on these large-eddy models, one could argue that if the eddies found in an agitated tank, generally of size proportional to the agitator diameter, are to be progressively reduced in size, one would expect the rate of mass transfer across the interface to decrease correspondingly. This is because as the reduction occurs, there will be few large eddies with sufficient kinetic energy to overcome surface tension and cause surface renewals. Furthermore, if a series of size reduction is allowed, it is conceivable that the gas-liquid interface would eventually become so calm that it could be considered as a laminar layer of liquid. For mass transfer across a laminar film of liquid, $k_c$ has been found to be very nearly independent of Reynolds number (Fortescue and Pearson, 1967; Davies and Warner, 1969). Thus, the objective of this experiment is to establish whether the trend mentioned above, that is, $k_c$ should decrease with the scale of eddy and eventually becomes independent of Reynolds number, exists.

## EXPERIMENT

A schematic diagram of the apparatus used is shown in Figure 1. The glass vessel was 30 cm in diameter and fitted with four solid baffles, 2.54 cm wide. Three screen cylinders of dia. 2.5, 5.0 and 7.5 cm were placed in the tank, concentric with the stirrer shaft. They were made from woven screens of 6 mesh for the second set of runs and of ten mesh for the third series of runs. The tops of these cylinders were slightly above the water surface.

The stirred tank was covered with a plexiglass plate with openings for gas inlet and outlet, impeller shaft, and sampling. The impeller, located 10 cm above the tank bottom, was a six-blade, bronze turbine, 10 cm in diameter, and was driven by a variable-speed motor. The blades were 2.5 cm in length and 1.9 cm in height.

After a series of pressure reduction, the gaseous $CO_2$ was fed to the tank through a long copper coil. The coil served as a temperature regulating device. Distilled water was the other fluid used; its temperature was not controlled but was reasonably constant at the room temperature of 20°C.